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# Social Consumer Agents in an Integrated Markets Model

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**Abstract**

The impact of cognitive and socially bounded consumer agents on an integrated markets model is investigated. The market scenario consists of a financial market with trading agents and a consumer market. The markets are coupled via learning production firm agents offering their products and shares for sale. The consumer agents are embedded in a social structure based on "small-world network" principles. The cognitive model of the consumer agents enables them to make their decisions according to the behavior of the adjacent social neighborhood and based on the degree of satisfaction and uncertainty they are facing. The potential and limitations of the consumer agent model are explored by applying a recently introduced Markov chain Monte Carlo method. Therefore certain empirical phenomena or "stylized facts" are selected for reproduction within the simulation and the conditions of their occurrence are analyzed. It is shown that the properties of the social network structure and the sensitivity of the agents' cognitive decision making process (heuristics) contribute significantly or are, in fact, enabling the complex phenomena of Bass curves observed in consumer market scenarios. Furthermore the results indicate that the structural properties of the emerged social networks are stable and match real-life social networks. Moreover we show that the network structure has a strong impact on the development of market share. Thus we suggest the use of the social network descriptive parameters, which could be discovered empirically, as predictive factors for marketing forecasts.

# 1 Introduction

Neoclassical economic theory is based on the assumption of rationally acting individuals, who are able to consider all available information in the decision-making process. As an early critic of economic agents with unlimited information processing capabilities Herbert Simon (1955, 1982) suggested the term “bounded rationality” to describe a more realistic approach to cover human problem solving. Indeed, the complexity of human behavior suggests that a choice model should explicitly capture uncertainty. Real economic agents are restricted at least in their cognitive (knowledge) and computational abilities (Mullainathan and Thaler, 2000). Enriched by a social network perspective, which states that most behaviors are also closely embedded in networks of interpersonal relations, an additional focus lies in the relationships among interacting units. According to Wassermann and Faust (1994) a social network is a set of people or groups of people (“actors” or agents) with certain pattern of interactions (“ties”) between them. Central concepts are:

- actors and their actions are viewed as interdependent
- relationships among actors are channels for transfer of resources
- the network structure provides constraints and opportunities for individual action
- lasting patterns of relations are conceptualized as structure.

Recent work on social networks has focussed on distinctive features of network structure (Newman et al., 2002). One of these is the “small world” effect first described by Milgram (1967). His experiment involved letters that were passed between pairs of apparently distant people. Milgram found that the typical chain from acquaintance to acquaintance only has a length of about six persons (popularly known as “Six Degrees of Separation”). Since then dozens of academic studies have revealed that many networks have related “small-world” properties (see for example Watts and Strogatz, 1998). Usually the topology of a (social) network is assumed to be either completely regular or completely random. However, many biological, technological and social networks lie between these two extremes. These systems are highly clustered, like a regular lattice, but have small path lengths, like random graphs and are named “small-world” networks. From a social systems perspective this means that it only takes a small number of well-connected people to make a world small (Collins and Chow, 1998).

In this article we introduce an agent-based computational economic model, which incorporates boundedly rational agents embedded in a social network structure. Computational economic models bridge the gap between theoretical and empirical economics. They can represent a testbed, which enables us to investigate the predictions of a theory under conditions which are too complex to be addressed analytically. Hence computational models can be used to gain insights into complex systems and furthermore suggest new hypotheses to be tested in empirical studies (for a review of agent-based computational economics see Tesfatsion, 2002). We present a considerably extended version of the integrated markets model, recently introduced by Sallans et al. (2002, 2003). The model spans two markets: a consumer market and a financial equities market. The consumer market consists of production firms offering a good for sale, and customer agents who can purchase the good. The financial equities market consists of stock traders who can buy and sell shares in the production firms. The new model focuses on a more life-like model of consumer agents. The new agents are embedded in a social structure based on “small-world” principles and incorporate an enhanced cognitive decision structure related to the consumat approach presented by Janssen and Jager (2000). Since in real life people do not behave in a systematic manner (see for example Gintis, 2000) a rational agent approach can not account for behavioral dynamics such as habits, imitation and social comparison. To explore how such behavioral dynamics affect the evolution of an economic system, it is practical to apply a more sophisticated approach in the integrated modeling context. The main contribution of this approach is that it increases the psychological richness and possibilities of validation of the simulated behavioral dynamics since it introduces behavioral rules based on a conceptual meta-model of behavior. This will take account of certain types of behavior like imitation, social comparison and market dynamics like lock-in, loyalty and bandwagon or snob effects.

We evaluate the fruitfulness of the new approach by comparison of the model’s output (macro level, for example a firm agent’s market share) to known “stylized facts” in consumer and financial markets. Stylized facts are robust empirical phenomena, which characterize market dynamics and have been observed in real markets. For the validation of our integrated markets model we use a well known stylized fact found in consumer markets, the Bass curve. It is described by the Bass diffusion model and was introduced by Frank M. Bass (1969) in his now classic paper.

## 2 The Integrated Markets Model

The model consists of two interacting markets, a consumer and a financial equities market. The consumer market simulates the manufacture of a product by *production firms*, and the purchase of the product by *consumers*. The financial market simulates trading of shares. The shares are bought and sold by *financial traders*. The two markets are coupled: The financial traders buy and sell shares in the production firms, and the managers of firms are concerned with their share price. The traders can use the performance of a firm in the consumer market in order to make trading decisions. Similarly, the production firms can potentially use positioning in product space and pricing to influence the decisions of financial traders (see figure 1).

The simulator runs in discrete time steps. Simulation steps consist of the following operations:

- Consumers make purchase decisions
- Firms receive an income based on their sales and their position in product space
- Financial traders make buy/hold/sell decisions. Share prices are set and the market is cleared
- Every $N_p$ steps, production firms update their products or pricing policies based on performance in previous iterations

The integrated markets model is intended to be a generic model of the interaction between financial and consumer markets. It has been shown to reproduce a large range of empirical “stylized facts” including learning-by doing in the consumer market; low predictability, high kurtosis and volatility clustering; and correlations between volatility and trading volume in the financial market.

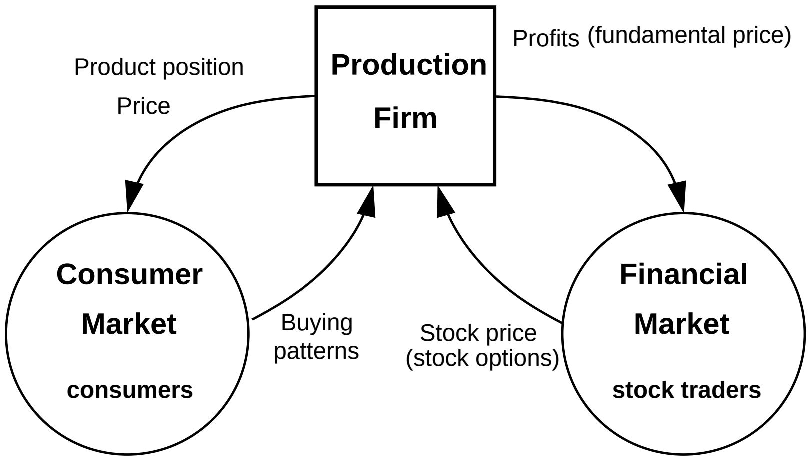

Figure 1: The Integrated Markets Model. Consumers purchase products, and financial traders buy and sell shares. Production firms link the consumer and financial markets, by selling products to consumers and offering their shares in the financial equities market (from Sallans et al., 2003).

### 2.1 The Consumer Market

The consumer market consists of firms which manufacture products, and consumers who purchase them. The consumers will re-purchase at regular intervals. The product space is represented as a two-dimensional simplex, with product features represented as real numbers in the range [0,1]. Each firm manufactures a single product, represented by a point in this two-dimensional space. Consumers have fixed preferences about what kind of product they would like to purchase. Consumer preferences (individual needs) are also represented in the two-dimensional product feature space. There is no distinction between product features and consumer perceptions of those features. Each consumer agent is embedded in a social structure which influences its social needs and incorporates a cognitive decision structure which accounts for its committed behavior (repetition, imitation, social comparison, deliberation). Consumer agents react to their individual needs, social needs and the price of the produced products. We describe the details of the consumer agents in section 3.

### 2.2 Production Firms

The production firms are adaptive learning agents. They adapt to consumer preferences and changing market conditions via a reinforcement learning algorithm (Sutton and Barto, 1998). In each iteration of the simulation the firms must examine market conditions and their own performance in the previous iteration, and then modify their product or pricing. A boundedly rational agent can be subject to several kinds of limitations. These limits manifest themselves in the firm's representation of its environment and its knowledge of its competitors. The firms do not have complete information about the environment in which they operate. In particular, they do not have direct access to consumer preferences. They must infer what the consumers want by observing what they purchase. Purchase information is summarized by performing "k-means" clustering on consumer purchases. K-means is a common clustering technique used in consumer market research. The number of cluster centers is fixed at the start of the simulation. The current state information consists of the positions of the cluster centers in feature space, along with additional state information such as whether or not the previous action was profitable or boosted stock price, and where the competitors products are located. This information gives a summary of the environment at the current time step.

Firms make decisions based on a finite history of states of some length. This limited history window represents an additional explicit limit on the firm's knowledge. In each iteration the firms can take one of several actions. The actions include taking a random action, doing nothing, raising or lowering product price, or moving the product in feature space. The random action was included to allow the firm to explicitly choose to take a “risky” exploratory action. A firm's manager seeks to modify its behavior so as to maximize an external reward signal. This reward signal can be viewed as the managers compensation for its actions. Given the reward signal, the firm learns to make decisions using a reinforcement learning algorithm (Bertsekas and Tsitsiklis, 1996; Sutton and Barto, 1998). Given the reward signal at each time step, the learning agent attempts to act so as to maximize the total (discounted) reward received over the course of the task. The discounting indicates how “impatient” the manager is to receive its reward. It can also be related to the interest rate for a low-risk investment or the rate of inflation.

## 2.3 The Financial Market

Our financial market represents a standard capital market model (see for example Arthur et al., 1997; Brock and Hommes, 1998; Dangl et al., 2001). Myopic investors maximize their next period's utility subject to a budget restriction. At each time step agents invest their wealth in a risky asset (a stock or index of stocks) and in bonds, which are assumed to be risk free. The risk free asset is perfectly elastically supplied and earns a risk free and constant interest rate. Investors are allowed to change their portfolio in every time step. As in many other heterogeneous agent models we assume that two kinds of investors exist: Fundamentalists and chartists. The two types of investors differ in how they form expectations of future prices. Additionally investors have different time horizons which are modeled via the time length agents look back into the past. Fundamentalists determine their price expectations according to a model based on fundamental information, which in our model are past dividends. They calculate a fair price and expect that the current price will gradually move towards it at some fixed rate. A fundamentalist assumes that the fair price is a linear function of past dividends. Chartists use the history of the stock prices in order to form their expectations. They assume that the future price change per period equals the average price change during the previous periods. The market uses a sealed-bid auction, where the clearance mechanism chooses the price at which trading volume is maximized. Note that there may be a range of prices that would maximize volume. We select the maximum price in this range. If there are buy orders but no sellers then the share price is set to the maximum bid. If there are only sell orders then the price is set to the minimum ask. If there are no orders in a time period, then the price remains unchanged. Each trader specializes in a single firm, and only buys or sells shares in this firm. Each trader is initialized with a supply of shares in its firm of interest.

Let us have a look at the timing of the events within the financial model. The first step is the formation of expectations. Based on past prices and dividends an investor forms its expectation about the distribution of the next period's price and dividend. The trading agent is then able to determine the demand function, which is submitted to the stock market via limit buy orders and limit sell orders. After the orders of all agents are submitted the stock market calculates this period's equilibrium price At the end of the period the current dividend is announced and becomes public information.

# 3 Social Consumer Agents

The integrated markets model, which already incorporates a validated consumer and financial market, will serve as a testbed for our new consumer market. The advantage of this approach is that we profit from a validated and realistic financial market while we improve the consumer market model. It allows us to investigate the behavior of the new bounded rational and socially connected consumer agents in an integrated context.

The consumer market consists of product manufacturing firm agents and regularly re-purchasing consumer agents. During a simulation time step, each consumer must make an individual product purchase decision based on the following factors:

- its preference in product space (individual needs)
- the behavior of its social network and
- the current price of the offered products.

Furthermore the agents are able to commit to repetition, imitation, social comparison and deliberation behavior dependent on their cognitive state (satisfaction and uncertainty).

## 3.1 Consumer Preferences

The product features are represented in two dimensions as pairs of real numbers in the range [0,1]. Each firm manufactures a single product with certain properties, which define the product's position in feature space and are adaptable to the consumer's demands. Each consumer agent is initialized with a random product preference in product feature space. There is no distinction between product features and consumer perceptions of those features since we were not concerned with advertising (for an example of a consumer markets model using this distinction see Buchta and Mazanec, 2001).

The product preference *IN* represents the individual needs of an agent (equation 1). It is calculated each simulation time step and is a function of the distance between the firms' manufactured products and the consumer agent's own preferences. The measure is computed as one minus the Euclidian distance between the position of the ideal preferred product of customer *c* ($IP_c$) and the position of the produced product *i* ($PP_i$) in the two-dimensional feature space (equation 1).

$$\mathrm{IN}_{c,i} = 1 - \sqrt{\frac{(PP_i - IP_c)^2}{2}} \tag{1}$$

## 3.2 Social Networks

Every consumer agent is embedded in a social network structure which is randomly initialized regarding the number of neighbors and the topology of the network.

For a social network structure to have "small-world" topology it must exhibit certain properties. This can be easily described in a graphical example. Figure 2 shows three examples of networks with fifteen consumers, each with an average of four neighbors. Every vertex represents one consumer agent and an edge represents a bi-directional connection between two consumer agents. The left picture shows a completely regular graph (random connection probability per consumer is zero), while the right graph represents a completely random connected topology (random connection rate is one). Although regular networks and random graphs are useful idealizations, many real networks lie between the extremes of order and randomness. For intermediate values of randomness (the middle picture consists of fifteen percent random connections) the graph can be interpreted as a small-world network. To construct small-world network topologies we start out with a completely regular graph. Then with a certain probability we reconnect each edge to a randomly chosen vertex over the entire ring, with duplicate edges forbidden. The small-world networks are much more clustered than a random graph. Hence if consumer A is linked to B and B is linked to C, there is a greatly increased probability that A will also be linked to C, a property that is called transitivity (Wassermann and Faust, 1994). Despite the high clustering small-world networks have characteristic small path lengths, like random graphs (Watts and Strogatz, 1998; Strogatz, 2001).

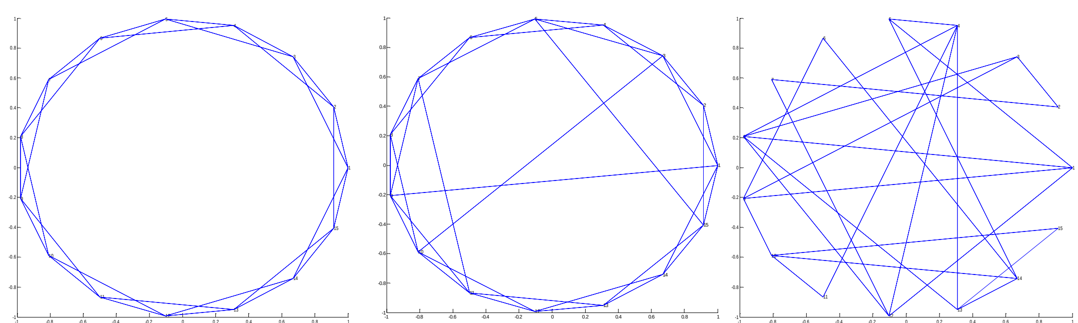

Figure 2: Example of a regular graph (left), small-world network (middle) and a completely random graph. Each graph is consists of fifteen consumers, all connected with on average four neighbors (adapted from Watts and Strogatz, 1998).

We define the "social" market share *SM* (equation 2) to transform the social network into a relevant decision structure for an individual consumer agent *c*. It is represented by the quantity of the last purchases of product *i* in the consumer agent *c*'s social neighborhood ($LPP_{c,i}$) divided by the number of all purchases occurred in its neighborhood (products range from one to *n*).

$$\mathrm{SM}_{c,i} = \frac{\sum_{c} LPP_{c,i}}{\sum_{c}\sum_{n} LPP_{c,n}} \tag{2}$$

Intuitively, the social market share represents a measure of a product's popularity amongst a "clique" of socially connected people.

### 3.3 Cognitive States

According to the consumat approach (Janssen and Jager, 2000) two intrinsic cognitive states can account for different types of behavior and decision making. Dependent on their experienced level of satisfaction (*S*) and uncertainty (*U*) consumer agents are able to commit to repetition, imitation, social comparison and deliberation behavior.

We define that consumer *c* experiences the following satisfaction level (*S*) regarding the purchase of product *i* (equation 3).

$$S_{c,i} = SM_{c,i} * SNW + IN_{c,i} * (1 - SNW) + (1 - \frac{P_i}{\max(P)}) * PSAT \tag{3}$$

Thus consumer agents can react to their individual needs (*IN*), social needs (*SM*) and the prices of the produced products (*P*) with modification of their cognitive parameter satisfaction (*S*). Furthermore satisfaction weighs the social market share (weight *SNW)* against individual needs (weight *1-SNW)* and the price of the offered product (weight *PSAT*).

A consumer agent's experienced uncertainty (*U*) is defined as the squared deviation of the actual level of satisfaction ($S_t$) from its expected level of satisfaction which equals the agent's last obtained satisfaction level ($S_{t-1}$, see equation 4).

$$U_t = (S_t - S_{t-1})^2 \tag{4}$$

To differentiate between possible actions threshold parameters for minimum satisfaction ($S_{min}$) and maximum uncertainty ($U_{max}$) are introduced. They also represent an agent's bias to commit to a certain category of action with a certain probability (table 1).

Table 1: Actions resulting from cognitive state variables of consumer agents (according to Janssen and Jager, 2000).

| Cognitive state | Satisfied | Not Satisfied |
|---|---|---|
| Certain | Repetition | Deliberation |
| Uncertain | Imitation | social comparison |

The agent's performed behavior and purchase decision is a result of its experienced levels of satisfaction and uncertainty:

- Repetition: if the agent experiences satisfaction ($S > S_{min}$) and is also certain about its choice (that means that its last choices nearly met its expectations hence $U \leq U_{max}$)

then it has no reason to change his last decision. Therefore the customer agent will consume exactly the same product which it purchased the last time step.

- Imitation: if a customer agent again feels satisfied ($S>S_{min}$) but it experiences uncertainty (its last choice deviated much from its expectations and $U>U_{max}$) then the customer will investigate its social neighborhood and give the product a try, which is consumed most by its friends. If there is more than one product one will be randomly selected among the most purchased products.
- Deliberation: if a consumer is not satisfied ($S\leq S_{min}$) and it is certain (its expectations were met, thus $U\leq U_{max}$) it will purchase the product with the highest overall satisfaction value (according to equation 3). Again if there is more than one candidate product, one will be randomly selected among the most satisfying products.
- Social comparison: if the consumer agent happens to be not satisfied ($S\leq S_{min}$) and uncertain ($U>U_{max}$) the same time step, it will engage in a behavior called social comparison. This means that the agent will consider the product that is consumed the most in its social neighborhood (analogue evaluation of the social market share) but one that also exceeds or reaches its expectations for satisfaction (see equation 3) originating from his last consumption. If there is more than one candidate product, one will be randomly selected from the eligible products.

With this cognitive decision structure implemented and the agents' ability to relate their expectations to their social network we validate our simulation results against a complex behavioral phenomena and an empirically stable stylized fact found in consumer markets.

# 4 Model Validation

Gaining crucial insights into underlying mechanisms of real markets is a major goal of agent-based economic modeling. Thus a useful model should be able to reproduce observable market behavior or so-called "stylized facts" capturing the dynamics of the investigated market. Therefore we validate our model against these empirical properties by the use of a recently introduced algorithm based on Markov chain Monte Carlo (MCMC) sampling (Sallans et al., 2003).

The MCMC sampling helps us focus computational power on parameter space areas where stylized facts are reproduced very well. The goal is to understand the impact of parameters on model behavior, especially in these interesting areas. The stylized facts of the consumer market on which we mainly focussed were the properties of the consumers' social networks (sections 3.2 and 6) and the Bass diffusion model (sections 4.2). In order to quantify how well the market reproduces a stylized fact, an energy function is defined. The energy function represents a measure of the fit of the stylized fact to the output of the model. We construct an energy function for a stylized fact such that low energy corresponds to good reproduction of the fact. For example, an energy function for the Bass diffusion model would generate low values if a Bass curve fits very well to a firm's market share data (section 4.3).

The MCMC procedure first randomly changes model parameter values before a simulation run which generates one sample of model parameters. Then the quality of the generated parameter sample is evaluated based on a previously defined energy function which is unique for each stylized fact. The sample is accepted or rejected based on the energy and the MCMC procedure starts over until an arbitrary number of parameter samples is drawn (we choose a minimum of one thousand samples to get statistically significant results). The advantage of this method is that computational resources are distributed on what are probably the most interesting parameter combinations. The whole validation procedure works as follows:

- Selection of an empirical stable stylized fact
- Design of an adequate energy function for that fact
- MCMC simulation runs
- Analysis and perhaps repetition of simulation runs

The MCMC sampler we use was recently introduced by Sallans et al. (2003) and is based on principles of the Metropolis algorithm (Metropolis et al., 1953). It has the property that samples are more likely to be drawn from low-energy areas. The sampler acts as a "directed" random walk through model parameter space, avoiding high-energy areas. In the limit, parameter samples are drawn according to the normalized probability distribution defined by the energy function. But even without theoretical guarantees on the distribution of sampled parameters, the sampler can find good model parameter settings, and reveal interesting correlations between model parameters. In practice, we will not generate Markov chains which are sufficiently long to reach the equilibrium distribution. Instead we are content with one thousand samples drawn for each model run. While this is too short to allow for convergence, we can still examine the sample set to identify regions where stylized facts are well reproduced, and look for statistically significant correlations between parameters. Validation results for the Bass model runs are shown in section 5 and for the social networks in section 6.

### 4.1 Model Parameters

The focus of state of the art modeling techniques is not just to cover every market phenomena observed. Rather it lies on "noncritical" abstraction and careful parameter selection by gradually adding complexity once the previous model has been fully understood. This prevents the modeler from introducing ad hoc parameters to capture important causal relationships.

We are building on the foundation of the validated integrated markets model including consumer, firm and stock trading agents. Thus we started out with the originally given parameter values by Sallans et al. (2003) which guarantee a well functioning integrated financial and consumer market. Despite our goal to keep the model as simple as possible, additional parameters were necessarily introduced to account for the social network functionality and the improved agents' cognitive decision structure (table 2). All parameter values must be initialized before a model simulation is run. The column "value" of table 2 shows the start values used for the validation runs with the MCMC sampler. These values were found to be plausible based on evaluations of initial trial simulation runs (values in italics are given by the original model and were held fixed for all simulation runs).

Table 2: Model parameters for the integrated markets simulation.

| Parameter | Description | Range | Value | Reference |
|---|---|---|---|---|
| NCons | Number of simulated consumer agents | N | 100 | Section 3.2 |
| NNbs | Number of average neighbors per consumer agent | N | 4 | Section 3.2 |
| PClus | Percentage of randomness of small-world network | [0, 1] | 0.1 | Section 3.2 |
| | | | | |
| SNW | Weight of social network for satisfaction | [0, 1] | 0.5 | Equation (3) |
| PSat | Weight of price for satisfaction | [0, 1] | 0.5 | Equation (3) |
| $SAT_{min}$ | Threshold for minimum satisfaction of consumer agent | [0, 1] | 0.5 | Section 3.3 |
| $UNC_{max}$ | Threshold for maximum uncertainty of consumer agent | [0, 1] | 0.5 | Section 3.3 |
| $\nu$ | Firm learning rate | $R \geq 0$ | 0.001 | Sallans et al., 2003 |
| $\gamma$ | Reinforcement learning discount factor for firm | [0, 1] | 0.83 | " - |
| | | | | |
| $\alpha_\phi$ | Strength of profitability reinforcement to firm | [0, 1] | *0.47* | " - |
| $\alpha_p$ | Strength of stock price reinforcement to firm | [0, 1] | *0.53* | " - |
| N | Number of consumer cluster centers | N | *3* | " - |
| $N_f$ | Proportion of fundamentalist traders | [0, 1] | *0.57* | " - |
| $N_c$ | Proportion of chartist traders | [0, 1] | *0.43* | " - |

The quality of reproduction of the stylized facts should simply depend on the characteristics of the model's behavior. The parameters, which account for different features of our integrated markets simulation, can be grouped as follows:

- Social network properties: These are described by the number of consumers (*NCONS*), the average number of neighbors (*NNBS*), and the proportion of clustering (*PCLUS*). *NCONS* and *NNBS* account for the dimension of the artificial consumer market. The proportion of clustering (*PCLUS*) accounts for the complexity of the social network structure. While a value of zero represents a completely regular graph with low complexity, a value of one indicates a completely random connected topology consisting of the highest possible structural complexity (see figure 2 of section 2.2.1). For values between these extremes, the consumers' social structure exhibits small-world properties.
- Consumers' cognitive behavior: The consumers' behavior and decisions are triggered by the parameter weight for social needs (*SNW*), individual needs (*1-SNW*) and product price (*PSAT)*. These parameters account for the level of satisfaction and uncertainty experienced by the consumer. Furthermore thresholds for minimum satisfaction ($SAT_{MIN}$) and maximum uncertainty ($UNC_{MAX}$) will influence the action a consumer agent commits to (section 3.3).
- Firms' learning behavior: The firms' learning is triggered by the firm learning rate ($\nu$) and the reinforcement learning discount factor ($\gamma$). In order to get good learning, the firm learning rate ($\nu$) should be rather low. If the discount factor ($\gamma$) is low, the firm focuses on near-term, if it is high, it will focus on a long-term time horizon.
- Fixed parameters: The firm agent's parameters $\alpha_\phi$ and $\alpha_p$, which sum to unity, trade off the relative importance of profits and stock price in a firm agent's decision-making process (see section 2.2). *N* denotes the number of cluster centers as described in section 2.2. As mentioned in section 2.3 the stock market consists of fundamentalists and technical traders. The parameters proportion of fundamentalists ($N_f$) and proportion of chartists ($N_c$) maintain the heterogeneity of the market traders, which is necessary to preserve financial market liquidity and trading volume.

## 4.2 The Bass Diffusion Model

The seminal work of Frank M. Bass (1969) describes a simple mathematical model of market penetration of a new product or concept as a function of internal (for example word of mouth) and external influences (for example advertising). The model and its variations have been successfully applied by marketing scientists in many different areas for over 30 years. Examples include DirecTV (early 1990s), a satellite television service which forecasts new subscription rates, and RCA (mid 1980s), which effectively used an extension of the Bass model to forecast the sales of CDs as a function of the sales of CD players. Fields of application are usually the quantification of the speed of diffusion of durables and non durable products and the forecast of future consumer adoptions (see Van den Bulte, 2002, for a meta-analysis of research on different product types over different regions).

The diffusion of innovations is influenced by interpersonal and mass media communication. The effects of interpersonal communication in particular are thought to be a key factor for the speed and shape of the diffusion of an innovation (Rogers, 1983; Mahajan et al., 1990). The theory of network externalities provides a related explanation and quantification of increasing consumer demand and S-shaped diffusion of network goods or service sales over time (Grajek, 2002). Positive network externalities are defined as utility, which consumers derive from consumption of a good or service, increases with the number of other consumers. Economic literature usually distinguishes between direct and indirect network externalities (see for example Katz and Shapiro, 1985; Economides, 1996). Direct network externalities are related to physical networks (for example telecommunication technologies). The utility, which consumers derive from using these technologies, depends undoubtedly on the number of other users. An obvious reason for a positive dependence is that a larger network allows consumers to satisfy more communication needs and may decrease the common costs of the service. Another explanation might be the bandwagon effect since conspicuous consumption gives rise to a conformistic behavior (Leibenstein, 1950). A negative dependence between network size and consumers' utility might be justified by congestion or by non-conformism of consumers (snob effect). Indirect network externalities apply if a good consists of two complementary components: for example hardware and software. The latter exhibits supply-side economies of scale (see Katz and Shapiro, 1985). Obviously the amount of users of the hardware platform determines the size of the market for software and furthermore enhances the utility gained by use of the hardware.

The original Bass model makes adoption a function of innovation and imitation effects. The theory details the characteristic empirically observed sigmoid pattern which levels off to a maturity level (see figure 3). The spread of an innovation in a market can be characterized by the Bass formula as a discrete time model (equation 5, see also Morris and Pratt, 2003).

$$X_t = p(M - X_{t-1}) + q\frac{X_{t-1}}{M}(M - X_{t-1}) \tag{5}$$

Where $X$ denotes the number of consumers who will adopt at time t, $M$ represents the market potential or the maximum number of people who will use the innovation. The parameters $p$ and $q$ provide information about the speed of diffusion. The coefficient of innovation $p$ describes the external influences and the coefficient of imitation $q$ describes the internal influences. A high value for $p$ indicates that the diffusion starts out quickly but also decreases fast. A high $q$ represents a slow diffusion process at first, which accelerates quickly afterwards (for example take-off is slower for non durables and products with competing standards that require heavy investments). In our model, we validate the different firms' market share time series against a cumulative discrete standard Bass function which gives the absolute number of adopted consumers at a certain point in time (equation 6).

$$X_t = X_{t-1} + p(M - X_{t-1}) + q\frac{X_{t-1}}{M}(M - X_{t-1}) \tag{6}$$

The market potential parameter $M$ was set to one, representing the maximum possible proportion of agents in our competitive consumer market environment. Examples of standard bass curves (values for $p$=0.03 and $q$=0.38 describe the basic Bass model) generated by the integrated markets simulation are shown in figure 3.

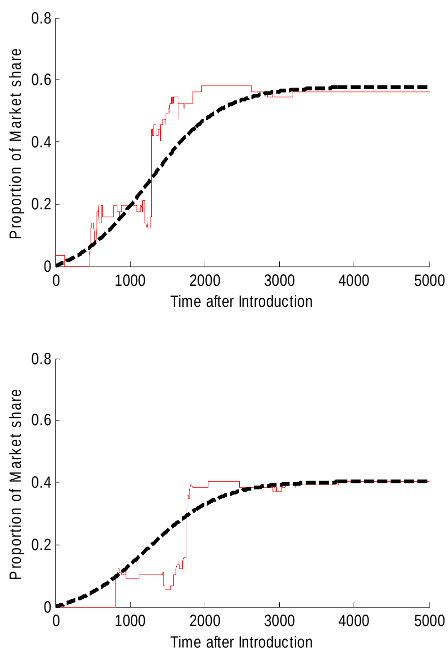


Figure 3: Examples of generated Bass curves in the artificial consumer market. The black dotted lines represents the standard Bass curve with the parameter values for $M$=1, $p$=0.03 and $q$=0.38.

## 4.3 The Energy Function for the Bass Model

To investigate which parameter settings have influence on the development of Bass curves in the artificial consumer market it is necessary to define an adequate energy function for the adapted Metropolis algorithm (section 4). Our purpose is to find a measure presenting the optimal fit of a standard Bass curve on the consumer market share time-series. The measure should neither depend on where in the data the Bass curve is located (translation invariance with respect to time) nor on the scaling of the curve (scale invariance, see for example Bishop, 1995). The crosscorrelation function (equation 7) represents a good solution to overcome these problems since the function is not sensitive to y-scaling (height) of the data, when comparing two different time series. To account for the x-scaling (time) the function is set up to compute the maximum correlation coefficient over *all* time lags (equation 8). Each data point of the sample (one discrete time step) equals a single simulation step.

$$\rho_{XY}(k) = \frac{\sigma_{XY}(k)}{\sqrt{\sigma_X \sigma_Y}} \tag{7}$$

$$\rho_{\max} = \max_k \{\rho_{XY}(k)\} \tag{8}$$

While $X$ denotes the market share time series of a certain firm in the artificial consumer market, $Y$ represents the time series of a standard Bass curve.

To find the optimal fitting standard Bass curve for $X$, we set up a nonlinear optimization algorithm based on golden section search and parabolic interpolation (see for example Forsythe et al., 1976; Hagan et al., 1996). The algorithm fitted standard Bass curve time series with different width (in $Y$) to $X$ and minimized the negative crosscorrelation over all lags between $X$ and $Y$. Hence the optimization algorithm varied standard bass curves by scaling until it found the maximum crosscorrelation coefficient (the best match). The energy for the MCMC sampler is then calculated as the reverse of the maximum correlation coefficient since low energy corresponds to good reproduction of the stylized fact (equation 9).

$$E = \frac{1}{\rho_{\max}} \tag{9}$$

## 5 MCMC Validation Results

The emerged market dynamics of the validated model and the identified set of parameter values where standard Bass curves could be reproduced very well are presented in the following sections. First we will describe the overall consumer market dynamics, followed by a detailed analysis of the parameters and their relationships grouped by their functionality. All our simulations were based on five firm agents (held fixed over all runs) acting in consumer markets initialized with one hundred consumer agents.

### 5.1 Overall Market Dynamics

Interestingly, the emerging market behavior of our simulation models is not restricted to the one investigated stylized fact (Bass curve) of a single firm. It is embedded in a realistic market scenario with oligopolic properties Empirical investigations have shown that in real-life markets it is very frequent to find oligopoly industries that are characterized by a large range of different market shares, with no two firms having the same market share. Traditional economic models of quantity competition oligopoly are not successful in explaining this stylized fact (Watt, 2002). The Cournot model predicts equal market shares for all competitors, while a generalized Stackelberg leader-follower oligopoly model with one leader, predicts a larger market share for the leader and equal market shares for all followers (see Stackelberg, 1934; Sherali, 1984; Daughety, 1988).

The aggregate market dynamics of our simulated consumer market is represented by the market share of each firm and reflects the empirically found oligopoly market related stylized facts. For example, one or more firms attracted certain consumers by successful introduction and development of their products, while the others lost in market share or engaged in price wars. Bass curves solely emerged in low energy areas of the defined Bass model energy function (figure 4) while they don't emerge in high energy areas (figure 5). The squared correlation coefficient $\rho^2$ (equation 8) gives the proportion of variance explained by the fitted Bass curves with respect to the market share data. It is useful to compare the Bass curve reproduction quality of the different models.

Figure 4 presents samples where Bass curves were well reproduced. Run 1 (left column) shows the market share of two competing firms. The market leader (firm 5) increasingly looses market share to the market entrant (firm 1) which introduced its innovation at the beginning of the time period. Run 2 shows a similar dynamic with four competing firms. Here firm 5 introduces its innovation and competes against firms 2, 3 and 4. The competition ends with two market leaders which basically divided the market. One is the former market leader the other winner is the innovative newcomer. Run 3 also presents one firm with an emerged Bass curve in its market share (firm 2). It competes against firm 1, the former market leader, and firm 5. The scenario also ends in a rather stable oligopoly.

The runs in figure 5 present samples from a high energy area of the Bass energy function which did not generate any Bass curves. Run 1 shows initially four competing firms (firm 1 to 4). As the theory of Stackelberg (1934) suggests the competition ends in a stable oligopoly with one leader (firm 3) with higher market share and two followers with a nearly equal market share (firm 1 and 2). Run 2 shows four competing firms with a rather oscillating market share. First firm 4 seems to be the market leader soon beaten by the newcomer (firm 2). Then firm 1 fights against 2 until firm 3 gathers the whole market share. Afterwards firm 1, 3 and 4 compete until firm 3 wins again. Simulation run 3 shows a rather soft competition where firm 2 and 5 increasingly gain in market share while the firms 1 and 4 seem to have a decreasing trend in market share.

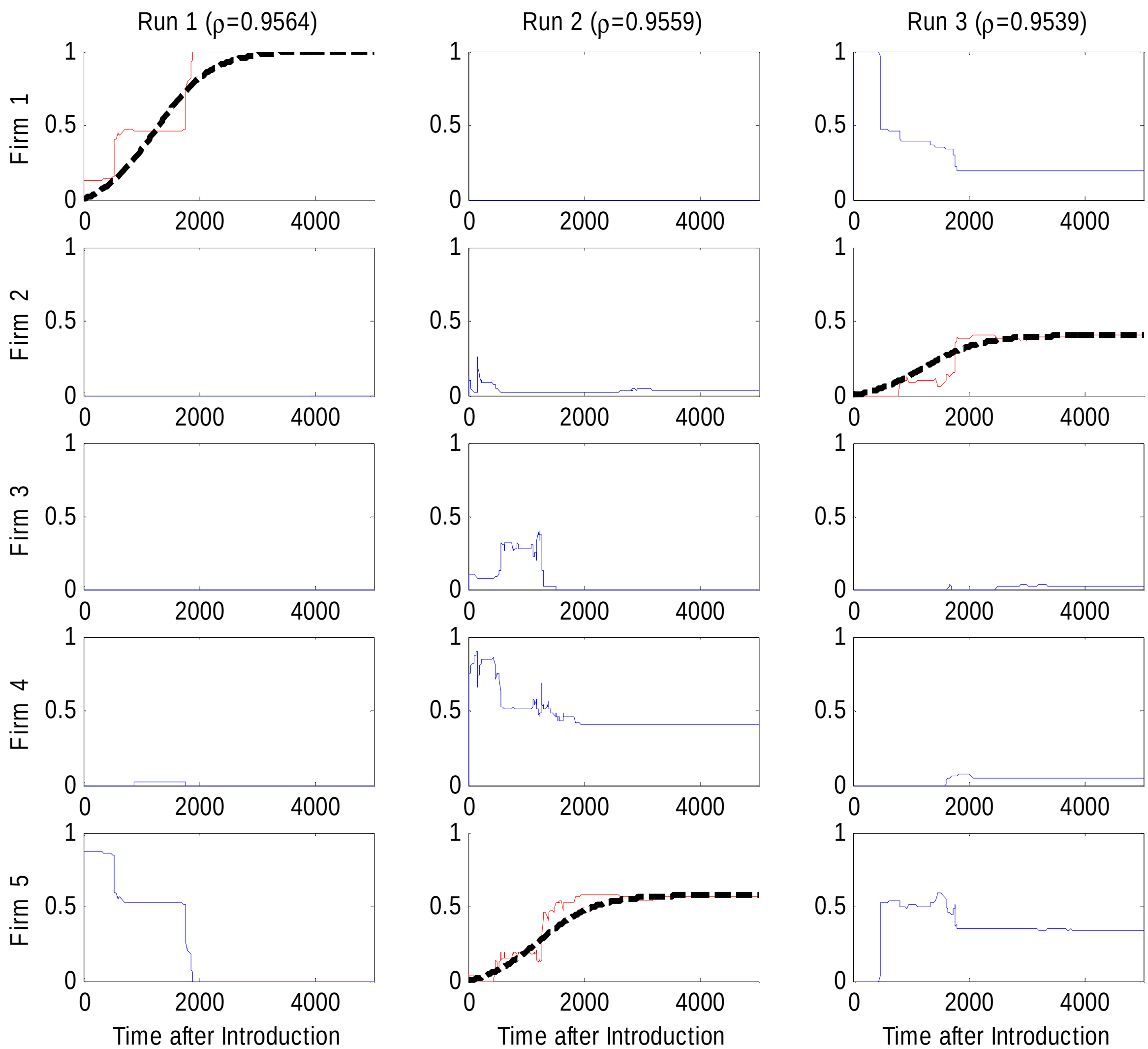


Figure 4: Three typical examples of simulation runs in low energy areas of the Bass energy function (section 4.3). Each column shows the emerged consumer market dynamics of an independent simulation run involving five firm agents and 69 (run 1), 57 (run 2), and 89 (run 3) consumer agents. The y-axis denotes the proportion of market share an individual firm agent could obtain at a specific point in time. The dotted line indicates the best fit of a standard Bass curve to the market share time series with a resulting correlation coefficient of ρ. The proportion of variance explained by the fitted Bass curves with respect to the market share data is given by a $\rho^2$ of 0.9147 (run 1), 0.9138 (run 2), and 0.91 (run 3).

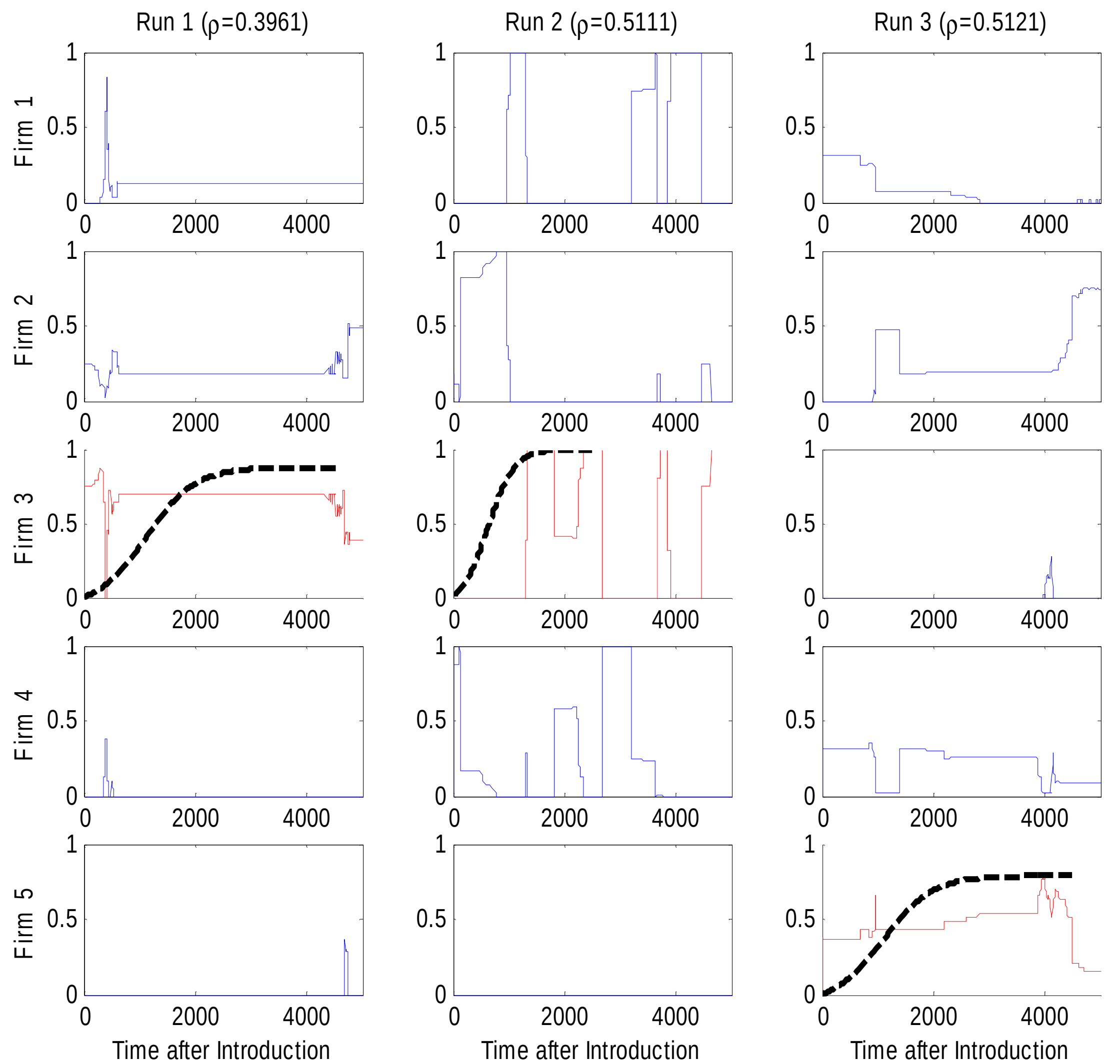


Figure 5: Three typical examples of simulation runs in high energy areas of the Bass energy function (section 4.3). Each column shows the emerged consumer market dynamics of an independent simulation run involving five firm agents and 74 (run 1), 75 (run 2), and 57 (run 3) consumer agents. The y-axis denotes the proportion of market share an individual firm agent could obtain at a specific point in time. The dotted line indicates the best fit of a standard Bass curve to the market share time series with a resulting correlation coefficient of $\rho$. The proportion of variance explained by the fitted Bass curves with respect to the market share data is given by a $\rho^2$ of 0.1569 (run 1), 0.2612 (run 2), and 0.2622 (run 3).

## 5.2 Ideal Model Parameters

The parameter values where standard Bass curves could be reproduced very well is presented in the form of histograms in figure 6. The “ideal” parameters don’t take on extreme values, which is an indicator for the plausibility of the model. Furthermore table 3 provides information about relationships between parameters. In the following sections we will describe and interpret the model parameters grouped by their functionality.

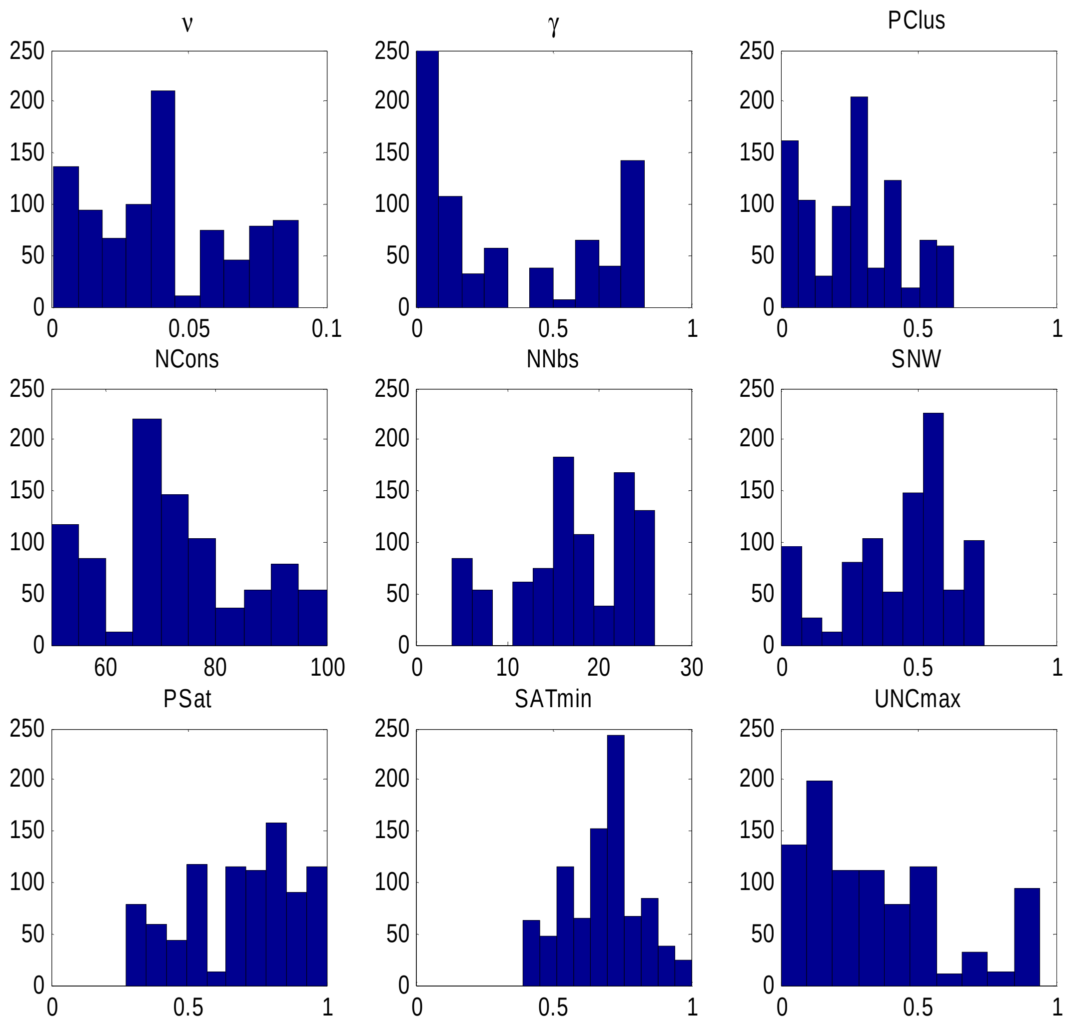


Figure 6: Histograms of parameter values from MCMC sampling for the Bass curve energy function. The histograms include the 90 % of samples with the lowest energy (equation 9).

Table 3: Correlation coefficients of the Bass validated integrated markets model.[1] The measures social clustering coefficient (*SCC*) and social clustering length (*SCL*) are described in section 6.

| | $\nu$ | NCons | $\gamma$ | NNbs | PClus | SNW | PSat | $SAT_{min}$ | $UNC_{max}$ | SCC | SCL |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu$ | 1.00 | | | | | | | | | | |
| Ncons | ***-0.74 | 1.00 | | | | | | | | | |
| $\gamma$ | 0.11 | -0.06 | 1.00 | | | | | | | | |
| NNbs | -0.09 | 0.03 | -0.37 | 1.00 | | | | | | | |
| Pclus | 0.43 | **-0.56 | -0.28 | 0.40 | 1.00 | | | | | | |
| SNW | -0.37 | **0.52 | -0.11 | -0.39 | -0.28 | 1.00 | | | | | |
| PSat | ***-0.76 | 0.49 | 0.01 | 0.21 | -0.35 | 0.19 | 1.00 | | | | |
| $SAT_{min}$ | ***-0.75 | 0.49 | -0.38 | -0.21 | -0.29 | **0.52 | **0.57 | 1.00 | | | |
| $UNC_{max}$ | 0.33 | 0.18 | 0.11 | -0.37 | -0.03 | 0.44 | -0.44 | -0.19 | 1.00 | | |
| SCC | -0.29 | 0.14 | 0.14 | 0.36 | **-0.51 | -0.33 | 0.31 | -0.15 | -0.46 | 1.00 | |
| SCL | -0.22 | 0.37 | 0.31 | ***-0.83 | ***-0.65 | 0.46 | -0.14 | 0.27 | 0.37 | -0.06 | 1.00 |

regression coefficients significant at the **5 % level, ***1 % level.

## 5.3 Firms Learning

The firms learning behavior is dependent on the firm learning rate ($\nu$) and the discount factor ($\gamma$). In order to get good learning, $\nu$ should be rather low. If the discount factor $\gamma$ is low the firm focuses on near-term, if it is high it will focus on a long-term time horizon. For the Bass model an intermediate value of 0.04 for $\nu$ seems to be most appropriate (the initial value was set to 0.001, see figure 6). $\gamma$ is initialized with a value of 0.83 and has it's peak around the rather low value of 0.06. Relationships between the firm learning and other parameters are interpreted as follows:

- Market complexity: The firm learning rate is negatively correlated with the number of consumers (*NCONS*) with a significant correlation coefficient of –0.74 (p=0.0015, see table 3), which indicates the necessity of better learning in a bigger and therefore more complex market environment (see figure 6, left picture).
- Product price: A significant negative correlation of -0.76 (p=0.001) exists between $\nu$ and the price weight (*PSAT).* Since the firms can change their product's price or its features this means that the importance of the product price for consumers increases if firms are able to engage in more intelligent actions, for example by making necessary price adaptations (figure 6, right picture).
- Consumer satisfaction and adaptation: Another finding is that the overall consumer satisfaction and uncertainty seems to decrease with the learning rate $\nu$. This is reflected by a negative correlation coefficient of –0.75 (p=0.0013) between $\nu$ and the threshold for minimum consumer satisfaction ($SAT_{MIN}$) and the positive trend (cc=0.33 but not significant with p=0.23) between $\nu$ and the threshold for maximum consumer uncertainty ($UNC_{MAX}$). Hence if the firms exhibit better learning ($\nu$ gets smaller) the consumers tend to be rather unsatisfied since the threshold for minimum satisfaction increased. They also get rather certain since the threshold value for

[1] Significance was measured in the following way: First, the sequence of parameter values was subsampled such that autocorrelations were insignificant at the one percent confidence interval. Given this independent sample, the correlations between parameters could be measured, and effective significance levels found.

uncertainty (*$UNC_{MAX}$*) increases. As an implication the consumer agents have a high probability to exhibit deliberative behavior, where they simply choose the maximum satisfying product (section 3.3). Thus they can react more sensitively to the firms' product price adaptations, a stylized fact which is strongly supported by the notion of network externalities (see section 4.2). Hence intelligent firm agents seem to lead to better adapted consumer agents in a Bass curve reproducing market scenario.

So far a good model for Bass curves seems to involve firms which are rather good learners, operating in a market environment of stable complexity and small world network properties (see also parameter *PCLUS* in the next section).

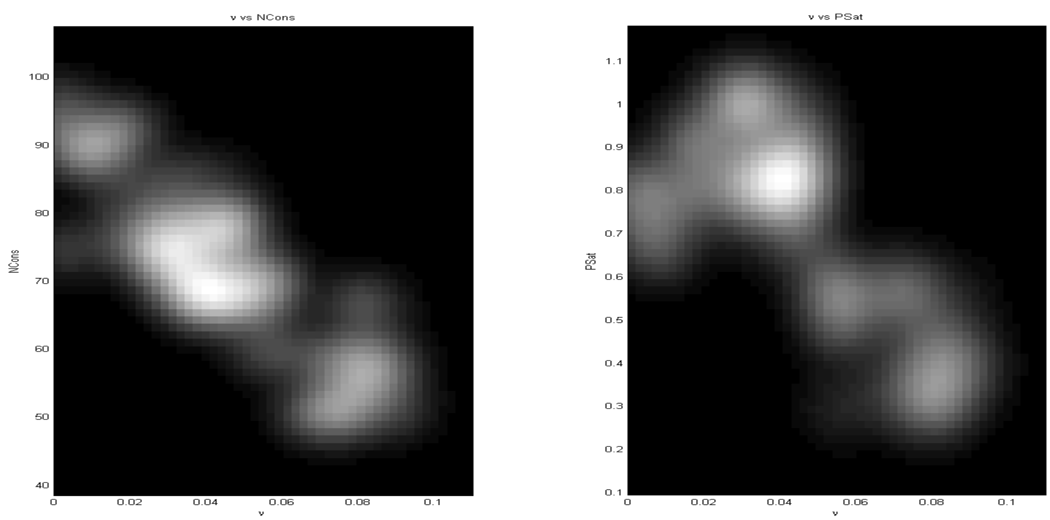


Figure 7: Negative correlation between firm learning rate (ν) and number of consumers (*NCONS*) and negative correlation between ν and price weight (*PSAT*). The plot shows the density[2] of samples for the different parameter values and includes the 90 % of samples with the lowest energy (equation 9).

## 5.4 Social Network Structure

The social network properties are described by the model parameters number of consumers (*NCONS*), average number of neighbors (*NNBS*), and proportion of clustering (*PCLUS*). *NCONS* and *NNBS* account for the dimension of the consumer market while *PCLUS* accounts for the complexity of the social network structure. As mentioned in section 3.2 a value of zero represents a completely regular graph with low complexity, while a value of one indicates a completely random connected topology. For values between these extremes, the consumers' social structure exhibits small-world properties (the exact properties of real-life social networks are described in section 6).

[2] The density plots were generated using the kernel density estimator for Matlab provided by C.C. Beardah at http://science.ntu.ac.uk/msor/ccb/densest.html (Beardah and Baxter, 1996).

A “good” social network for Bass curves seems to be one with a moderate number of consumers (peak at 70), with each of them having around 16 neighbors on average (figure 6). Social network related parameters are interpreted as follows:

- Small-world principles: The clustering rate had two major peaks, a smaller one at zero and one at a value which lies around 0.3. While the first represents a network with a regular topology the latter is a strong indicator for the preference of a social network based on small-world principles. But there is more evidence on the importance of small-world properties for the occurrence of Bass curves. From the first half of samples only 1.2 % exhibit a proportion of clustering bigger than 0.5. This is in contrast to the second half of samples where already 27.5 % show a *PCLUS* ≥ 0.5. Additionally we found that all samples with a *PCLUS* between 0 and 0.1 had an average Bass energy correlation coefficient (equation 8) of 0.699 (equals 49 % of explained variance). Samples with a *PCLUS* between 0.6 and 0.7 exhibited an average Bass correlation coefficient of 0.823 (equals 68 % of explained variance).
- Balanced network structure: A balanced social structure seems to be necessary for the Bass curve stylized fact. This is substantiated by a negative correlation of -0.56 (p=0.028, table 3) between number of consumers and proportion of clustering in the consumer market. Since the proportion of clustering accounts for the complexity of the social network structure an increased number of consumers (increased dimension and complexity) interestingly leads to the preference of lower proportion of clustering by the MCMC sampler.

Hence the consumer market seems more likely to reproduce Bass curves if the social network has balanced complexity and is structured like a small-world network.

### 5.5 Consumers’ Cognitive States

In our integrated markets model the consumers’ cognitive behavior and decisions are triggered by the parameter weight for social needs (*SNW*), individual needs (*1-SNW*), and product price (*PSAT)*. Thresholds for minimum satisfaction ($SAT_{MIN}$) and maximum uncertainty ($UNC_{MAX}$) regulate the actions consumers will most likely commit to (see section 3.3). Consumer agents’ cognitive parameters are interpreted as follows:

- Network externalities: Our simulation results support the hypothesis of positive direct network externalities as an underlying mechanism for Bass curves (section 4.2). First as mentioned in section 5.3 consumer agents’ sensitivity to price increases with more intelligent firm actions (price adaptations). In addition the level of minimum satisfaction increases with the weight of price (cc=0.57, p=0.028, see table 3). Hence if the price gets more weight the consumers tend to be rather unsatisfied since the threshold for minimum satisfaction increased. This is another indication for positive network externalities since consumers seem to become more satisfied with a lower price (weight). Furthermore the top 30 % of samples (sorted by reproduction quality of Bass curves $\rho_{max}$, see equation 8) exhibit a mean correlation coefficient $\rho_{max}$ of 0.75 and a mean consumer satisfaction proportion of 97 % (measured by the proportion of appearance of the consumer behaviors “imitation” and “repetition”, see also section 3.3). In contrast the first 70 % of samples with a mean Bass reproduction correlation coefficient $\rho_{max}$ of 0.67 show only 91 % consumers engaged in "satisfying" behavior. Hence there is a positive trend for increasing consumer satisfaction with quality of Bass curve reproduction. This can be explained by positive network externalities.

- Price vs. social needs: For the reproduction of standard Bass curves the best weighting factor for the social market share parameter (*SNW*) lies slightly above the initial value of one half (with a concentration around 0.6), while the price weight (*PSAT)* has its peak at a value of 0.8. This implies a normalized proportion for social needs of 0.33, individual needs of 0.22 and price of 0.44 (see equation 3). Thus for the satisfaction function of an individual agent the social market share slightly outweighed the individual needs, while the price seemed to be the most dominant factor. Although the latter finding is strongly supported by neoclassical economic theory which states that supply and demand both are function of price, Bass curves seems to need some additional cognitive and social parameters to occur in our model.
- Consumer satisfaction and uncertainty: The minimum satisfaction threshold ($SAT_{MIN}$) has a relatively high peak with the highest concentration around 0.7. This could mean that consumer agents in general are experiencing a very high level of satisfaction or they are rather committing themselves to a social comparison or deliberation decision style since the probability to be unsatisfied is rather high (section 3.3). The uncertainty threshold has its highest concentration at a rather low value (0.16), which indicates that consumer agents experience a rather low level of uncertainty in general or agents mostly engaging in repetition or deliberative behavior. In order to distinguish between these possibilities we will now analyze the consumer agents conducted actions.
- Consumer decisions: The Bass Model summarizes in a simple mathematical form the key finding from over 4,000 diffusion studies: most people wait until they have witnessed peers having favorable experiences with the technology or service before they adopt. Hence most people imitate rather than innovate. Analysis of our simulated consumer decisions reveals that the actual dominant consumer behavior is repetition with a proportion of 89.03 %, followed by deliberation (10.13 %), imitation (0.61 %) and social comparison (0.24 %) for the top 30 % of samples of the MCMC sampler (figure 8). Thus repetition behavior seems to be the most important mechanism for the emergence of Bass curves in our consumer market model. Since the consumer market of our integrated markets model is based on repeated purchases (every consumer purchases once at a simulation time step) the development of standard Bass curves over time heavily relies on consumer's repetition behavior. Hence the model shows behavior consistent with the Bass diffusion theory (section 4.2). Furthermore repetition can be viewed as a type of imitation behavior since the consumer agent imitates its own last decision. For commitment of repetition behavior consumer agents must experience satisfaction and certainty (low levels of uncertainty). For deliberation behavior they need to be certain and unsatisfied. Despite the high threshold for minimum satisfaction the consumer market environment consists of rather certain agents, who are switching between repetition (when they are satisfied) and deliberation behavior (when they are unsatisfied).

Our results show that the emergence of Bass curves in consumer markets can be explained by the underlying consumer agents' repetition and imitation behavior which leads to increasing demand, and deliberation behavior which refers to positive network externalities and leads to increased price sensitivity.

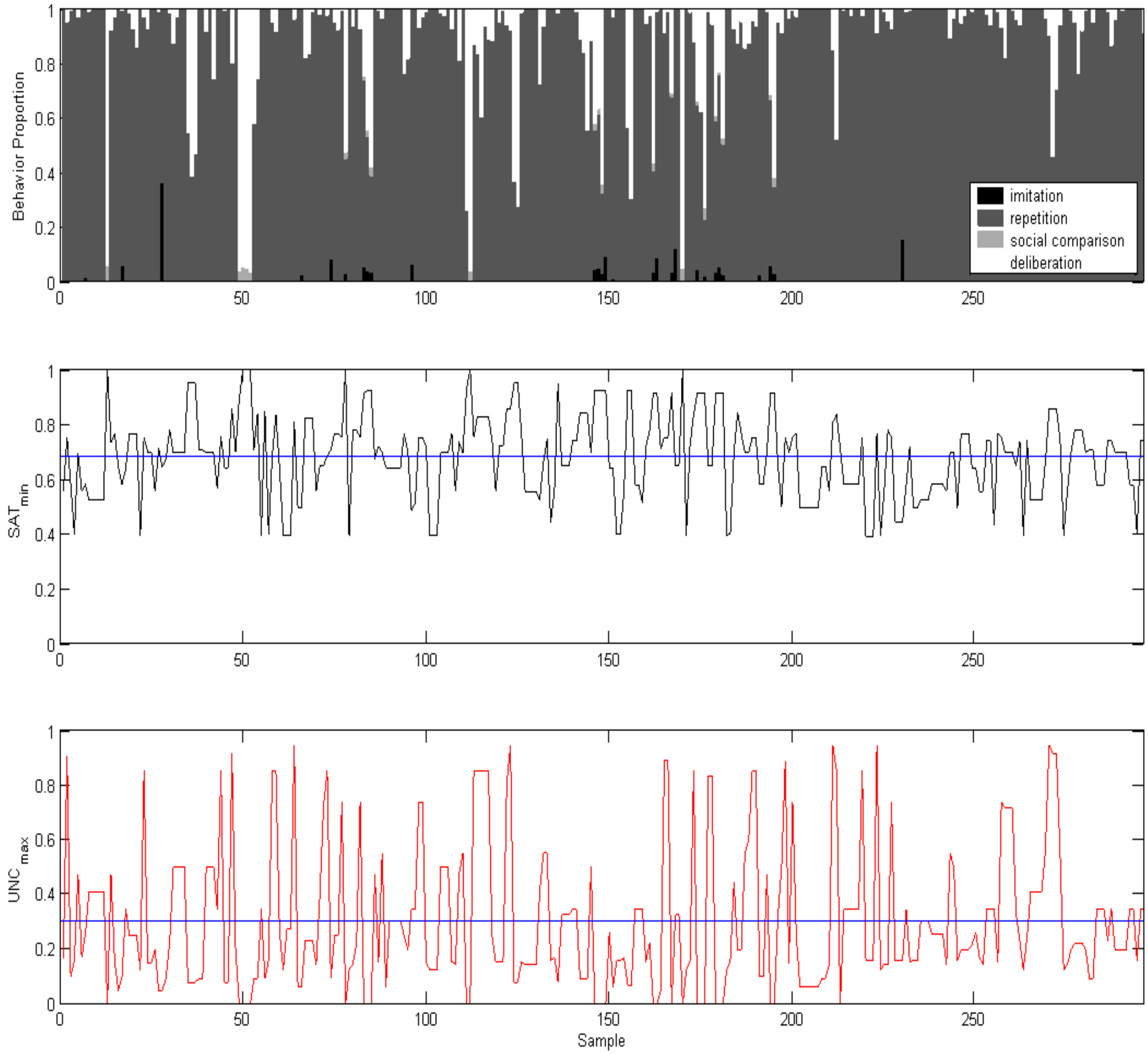


Figure 8: Consumer behavior separated into proportions of imitation, repetition, social comparison and deliberation behavior (upper diagram). The development of thresholds for minimum satisfaction (middle diagram) and maximum uncertainty (lower diagram) shows rather high levels for satisfaction and low levels for uncertainty as indicated by the mean of the values (the straight line). The plot includes the 30 % of samples with the lowest energy (equation 9).

## 6 Comparison to Real-Life Social Networks

In the previous section a small-world network topology was not explicitly imposed, but occured because it led to the generation of Bass curves. We implemented additional model runs, in which we forced the simulation to generate consumer markets with small-world structures related to a real-life social network. We denote this new model as the "optimized" model. The "optimized" model should exhibit a more life-like social network structure in the consumer market. From the comparison of the "normal" Bass validated model as already described in sections 4 and 5 and the "optimized" model we expected to gain supplementary insights into the role of the small-world properties for our consumer markets.

### 6.1 Social Clustering Coefficient and Characteristic Path Length

First we introduce two additional estimators to better characterize the social network structure. One is the characteristic path length (*SCL*) and the other is the clustering coefficient (*SCC*, see Watts and Strogatz, 1998).

The characteristic path length measures the typical separation between two agents in the network (a global property) and is defined as the number of connections needed for the shortest path between two agents, averaged over all pairs of agents in the consumer market. To find the shortest path between two agents the Floyd-Warshall algorithm was applied (see for example Cormen et al., 2001).

The clustering coefficient measures the cliquishness of a typical neighborhood (a local property) and is the fraction of existing connections compared to all possible connections within an agent's neighborhood, again averaged over all consumer agents. Suppose that the consumer agent c has $NN_c$ number of neighbors and $NC_c$ actual connections between them. Then its clustering coefficient $SCC_c$ is defined as follows (equation 10):

$$SCC_c = \frac{NC_c}{NN_c(NN_c - 1)\frac{1}{2}} \tag{10}$$

The characteristic path length (*SCL*) and the clustering coefficient (*SCC*) are both a function of the amount of randomness or complexity of the network structure (expressed by the parameter proportion of clustering *PCLUS*). Watts and Strogatz (1998) find that path length and clustering depend differently on the amount of randomness in the network. *SCL* decreases quickly, while *SCC* drops rather slowly with an increase in *PCLUS.* This can also be seen in table 3, where *SCL* and *PCLUS* exhibit a correlation coefficient of –0.65 (p=0.0091), while *SCC* and *PCLUS* show a cc of –0.51 (p=0.053). This leads to a small-world network with a high amount of clustering and a rather short characteristic path length. From a social systems perspective this means that it only takes a small number of well-connected people to make a world small (Collins and Chow, 1998). Watts and Strogatz (1998) lists an empirical example regarding these estimators derived from the Internet Movie Database and including approximately 90 % of the actors (table 4). Additionally they provide information about a randomly connected network with the same number of vertices (225 actors) and average number of edges (61 actors, see also table 4).

Table 4: Social network properties of movie actors Watts and Strogatz (1998)

| Social Network | Characteristic Path Length (*SCL*) | Clustering Coefficient (*SCC*) |
|---|---|---|
| Film actors | 3.65 | 0.79 |
| Random | 2.99 | 0.00027 |

### 6.2 The Optimized Model

To be able to compare our model results to a model with a more life-like social network structure we defined the "optimized" model.

We executed new model runs, where we excluded the parameter proportion of clustering (*PCLUS*) from being modified by the MCMC sampler. Hence the only social network related parameters to be manipulated by the sampler were the number of consumers (*NCONS*) and the average number of neighbors (*NNBS*). Each time the MCMC sampler changed one of these two parameters, we additionally ran a nonlinear optimizing algorithm (based on golden section search and parabolic interpolation, see for example Forsythe et al., 1976; Hagan et al., 1996) to find an optimal value for *PCLUS* for the current sample. This means that with a given *NCONS* and *NNBS* the optimizer was meant to manipulate the parameter *PCLUS* until it got as close as possible to the values for *SCL* and *SCC* shown in table 4. With this obtained "optimal" value for *PCLUS* we set up the new consumer market for the current sample. The value for *PCLUS* was retained for the following samples until the MCMC sampler again changed one of the two parameters *NCONS* or *NNBS*.

Hence the "optimized" model should explicitly exhibit small-world parameters similar to those shown in table 4.

### 6.3 Model Comparison

An overview of the differences between the optimized and the "normal" model is shown in table 5. Each of the values is discussed below:

- Values of the Bass correlation coefficient ($\rho_{max}$, see equation 8): Despite the fact that the means of both models look very similar in absolute values they are significantly different at the 5% significance level (p=0.0218) due to the low standard deviation. Interestingly there is a trend for the optimized model to have lower standard deviation. This is an indicator for the improved performance of a more life-like social network in showing the stylized fact Bass curves. Given more time and model runs we feel confident that the "normal" MCMC sampler would also find the superior solutions of the "optimized" model, which were found by the use of prior knowledge of social network structure.
- Characteristic path length (*SCL*): The "optimized" MCMC sampler with the Bass energy function seems to prefer social networks with a very stable path length of around 1.74. Since *SCL* measures a global property, the typical separation between two agents in the network, it also depends on the parameters *NCONS* and *NNBS* (section 6.1). Thus 1.74 was the closest value the optimizer could find under these given conditions. Interestingly these values are close to the "normal" Bass validated model which was not forced to generate small-world network properties. This substantiates the fact that the MCMC sampler in the "normal" model already

preferred more realistic social networks in order to increase the probability of the occurrence of Bass curves. Figure 9 (upper picture) shows the rather stable development of both parameters over time for the last 50 % of samples.

- Clustering coefficient or cliquishness (*SCC*): The mean values for *SCC* of both models are rather close again. Although the optimized model seems to develop its slightly but significantly higher average clustering coefficient (0.595) via a network structure using a higher average number of neighbors (~23) together with a lower proportion of clustering (0.10). The "normal" model seems to reach its neighborhood cliquishness via a higher proportion of clustering (0.26) but with a lower number of neighbors (~17). Since *PCLUS* and *SCC* are negatively correlated (cc=-0.51, p=0.053, see table 3) this is a consistent result. Figure 9 (lower picture) shows the development of the parameters over time. Although the mean values of both models are not that far off, the optimized model has a natural drift in the clustering coefficients towards its predetermined ideal value of 0.79 (from table 4, see figure 10 and 11).
- The social network properties are described by the number of consumers (*NCONS*), the average number of neighbors (*NNBS*), and the proportion of clustering (*PCLUS*). Interestingly both models are very close in their characteristic path length (*SCL*) and in their clustering coefficient or cliquishness (*SCC)*. But they seem to develop these properties in different ways. The optimized model develops a bigger market with more consumers and also more neighbors but exhibiting a lower *PCLUS* on average. The non-optimized model derives its properties for *SCL* and *SCC* by a higher *PCLUS* and lower number of consumers with a smaller neighborhood on average. This suggests that *SCC* and *SCL* may be a better measure for the description of social network structure than *PCLUS* alone and substantiates their usefulness. Given enough time and model runs we feel confident that the MCMC sampler would find all of these equivalent parametrizations.

Our results again support the importance of small-world network properties in consumer markets for the appearance of Bass curves. Interestingly this real-life observed topology emerges without being imposed explicitly in the "normal" Bass validated model.

Table 5: Comparison of the mean parameter values for the "normal" and "optimal" model including all samples (1000 per model). The values are shown with their 95 % confidence interval.[3]

| **Model comparison** | Mean | $\text{Mean}_{opt}$ | Std.Dev. | $\text{Std.Dev.}_{opt}$ |
|---|---|---|---|---|
| $\rho_{max}$ (equation 8)** | 0.744±0.014 | 0.747±0.011 | 0.2198 | 0.1824 |
| Characteristic Path Length (*SCL*)*** | 1.811±0.007 | 1.74 ±0.03 | 0.0752 | 0.3298 |
| Clustering Coefficient (*SCC*)*** | 0.411±0.009 | 0.595±0.009 | 0.1288 | 0.1371 |
| Number of Consumers (*NCONS*)*** | 73.28 ±0.87 | 89.24 ±0.76 | 14.08 | 12.26 |
| Number of Neighbors (*NNBS*)*** | 16.47 ±0.37 | 22.72 ±0.76 | 5.91 | 12.28 |
| Proportion of Clustering (*PCLUS*)*** | 0.261±0.011 | 0.104±0.007 | 0.1837 | 0.112 |

Mean differences are significant at the **5 % level, ***1 % level.

[3] The mean comparison is based on the nonparametric Wilcoxon ranksum test for independent samples.

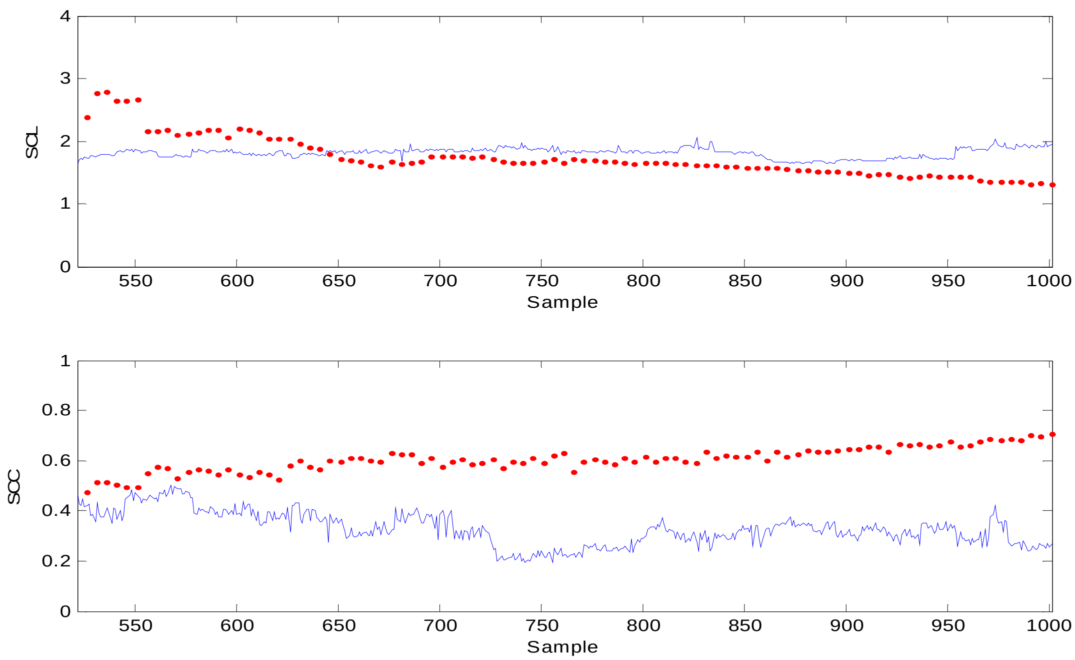


Figure 9: Development of the characteristic path length (*SCL*) and the social clustering coefficient (*SCC*) over time. The straight line represents the non-optimized model, the dotted line the optimized model values. The plot includes the last 50 % of samples of both models.

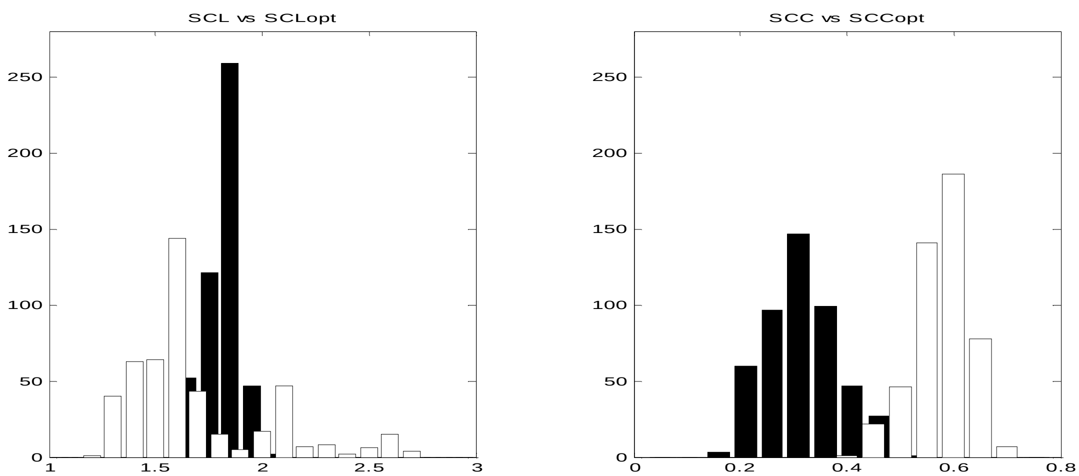


Figure 10: Comparison of the distribution of the characteristic path length (*SCL*, left) and the social clustering coefficient (*SCC*, right). The bars in the background represent the non-optimized model, the bars in the front the optimized model values. The plot includes the last 50 % of samples of both models.

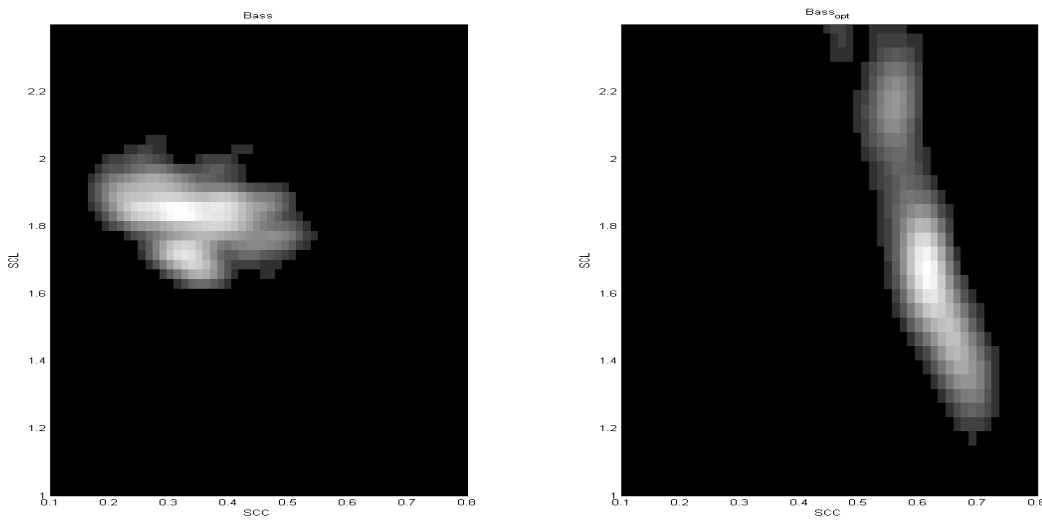


Figure 11: Comparison of the density plots of the characteristic path length (*SCL*) and the social clustering coefficient (*SCC*). The plot includes the last 50 % of samples of both models.

## 7 Market Share Forecasting

Our results so far have revealed the large impact of small-world network properties on the occurrence of Bass curves in our consumer markets (sections 5 and 6).

Because of this strong evidence we ran multiple linear regressions to test the hypothesis that the model parameters can be used as predictors for future market share data. For more detailed comparisons we additionally split the market share sample data into two and four parts. We treat the model parameters as predictors for the emerging future market share potential. In practice these parameters could be derived, for example, from investigation of the target market's social network topology. Table 6 shows the significant regression coefficients (model parameters) and the goodness of fit of the multiple regression (adjusted $R^2$) which gives the best measure of the proportion of variance explained by the predictor variables. Table 6 also shows the regression coefficients of the parameters vs. the average of the whole, the average of the halves, and the average of the quarters of the market share time series. The regression results indicate that the parameters proportion of clustering (*PCLUS*, section 3.2), weight for social needs (*SNW*, equation 3), and the maximum uncertainty threshold ($UNC_{MAX}$, section 3.3) were able to explain a significant part of the whole, the 1st half, and especially the 1st quarter of the market share. While it should be very easy for marketing practitioners to measure the complexity of the target consumer market's structural complexity (*PCLUS*), this might be rather difficult for social needs (*SNW*), and the uncertainty threshold ($UNC_{MAX}$). The latter could be discovered by, for example, surveys of consumer needs and consumer satisfaction.

Furthermore we ran multiple regressions where we used 80 % of our model parameters and market share time series data as training set and 20 % as a test set. While table 6 shows the overall results and could be used as forecasting tool applicable to real market share data, table 7 gives the results of our predictions from derived regression coefficients of the training set to the market share of the test set (out of sample forecast). Table 7 shows the regression coefficients and the goodness of fit of the regression (adjusted $R^2$) of the training set. $\sigma_{res}$ gives the standard deviation of the residuals from prediction of the test set data and S.E. the standard error of the residuals (we only used the significant coefficients for market share forecasting).

The multiple regression results again support the relevant role of the complexity parameter (*PCLUS*). In addition we show that the weight of social network for the consumer agents' satisfaction (*SNW*), the maximum uncertainty threshold ($UNC_{MAX}$), and the social clustering coefficient (*SCC*) seem to be consistent and substantial predictors for the emerging market share of our integrated markets model.

Table 6: Multiple linear regression of the integrated markets model parameters against different proportions of average market share. Bold values indicate significant regression coefficients.

| | Whole Market Share | 1st half | 2nd half | 1st quarter | 2nd quarter | 3rd quarter | 4th quarter |
|---|---|---|---|---|---|---|---|
| Adj. $R^2$ | 0.11 | 0.10 | 0.12 | 0.14 | 0.08 | 0.13 | 0.10 |
| $\nu$ | 0.62 | 0.51 | 0.73 | 0.31 | 0.72 | 0.63 | 0.83 |
| Ncons | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $\gamma$ | -0.23 | -0.22 | -0.24 | -0.35 | -0.10 | -0.13 | -0.34 |
| NN | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.01 | 0.00 |
| Pclus | ***0.43** | ****0.57** | 0.29 | ****0.70** | 0.44 | 0.35 | 0.22 |
| SNW | *****0.82** | *****0.88** | *****0.77** | *****0.95** | *****0.80** | *****0.76** | *****0.78** |
| Psat | 0.01 | 0.02 | -0.01 | 0.04 | 0.00 | 0.00 | -0.01 |
| $SAT_{min}$ | -0.10 | -0.17 | -0.03 | ****-0.29** | -0.05 | -0.01 | -0.05 |
| $UNC_{max}$ | *****-0.63** | *****-0.53** | *****-0.73** | ****-0.38** | *****-0.68** | *****-0.74** | *****-0.71** |
| SCC | 0.47 | 0.61 | 0.34 | 0.72 | 0.49 | 0.37 | 0.31 |
| SCL | 0.15 | 0.09 | 0.21 | -0.06 | 0.24 | 0.31 | 0.12 |

regression coefficients significant at the *10 % level, **5 % level, ***1 % level

Table 7: Multiple linear regression of the significant integrated markets model parameters against different proportions of average market share. Bold values indicate significant regression coefficients.

| | Whole Market Share | 1st half | 2nd half | 1st quarter | 2nd quarter | 3rd quarter | 4th quarter |
|---|---|---|---|---|---|---|---|
| S.E. | 0.12 | 0.12 | 0.12 | 0.12 | 0.13 | 0.12 | 0.10 |
| $\sigma_{res}$ | 0.24 | 0.26 | 0.24 | 0.31 | 0.26 | 0.24 | 0.25 |
| Adj. $R^2$ | 0.07 | 0.06 | 0.11 | 0.10 | 0.06 | 0.12 | 0.08 |
| Pclus | ***0.69 | ***0.80 | **0.58 | **0.86 | **0.75 | ***0.66 | *0.49 |
| SNW | ***0.76 | ***0.80 | ***0.73 | ***0.81 | ***0.80 | ***0.72 | ***0.75 |
| $UNC_{max}$ | ***-0.44 | *-0.32 | ***-0.57 | -0.11 | ***-0.54 | ***-0.59 | ***-0.55 |
| SCC | *0.97 | *1.15 | *0.79 | *1.20 | *1.09 | *0.83 | 0.75 |

regression coefficients significant at the *10 % level, **5 % level, ***1 % level

## 8 Conclusions

> Why is network anatomy so important to characterize? Because structure always affects function. For instance, the topology of social networks affects the spread of information and disease, and the topology of the power grid affects the robustness and stability of power transmission (Strogatz, 2001).

This paper explores the impact of a cognitive and socially bounded agent based consumer model on the integrated markets model recently introduced by Sallans et al. (2003). First we present a new consumer agent model (sections 2 and 3) which is embedded in a social structure based on “small-world network” principles (Milgram, 1967; Watts and Strogatz, 1998). Furthermore the agents follow a rather simple cognitive decision structure, but one which is able to account for valid behavioral dynamics such as habits, imitation and social comparison processes (Janssen and Jager, 2000). In the second part of the paper we explore the underlying mechanisms of Bass curves by validation of the model generated consumer markets data against empirically derived time series using a recently presented Markov chain Monte Carlo method (MCMC, see sections 4 and 5). The model produces consistent results as suggested by economic theory of network externalities. Our results show that the emergence of Bass curves in consumer markets can be explained by the underlying consumer behavior: repetition and imitation behavior, which leads to increasing demand, and deliberation behavior, which refers to positive network externalities and leads to increased price sensitivity (section 5.5). Furthermore a good model for Bass curves seems to involve firm agents which are good learners, operating in a market environment of stable complexity and small world network properties (sections 5.3 and 5.4).

The most striking fact that we document is the importance of small-world network properties for the occurrence and prediction of the Bass curves in consumer markets. Interestingly this real-life observed topology emerges in our model by selection of the MCMC sampler without being imposed explicitly (section 6). This has implications for marketing practitioners. Our results strongly suggest that it is useful to consider the structural properties of the target market, like cliquishness of the consumers’ neighborhood or complexity of the market’s social structure, and consumers’ cognitive parameters, like their (social) needs and consumer satisfaction, to improve the quality of sales forecasts. These results may even have more general applications than just in combination with the Bass model. Further (empirical) research seems to be fruitful and necessary regarding the small-world properties in conjunction with marketing forecasts. One line of research could focus on the grouping of the markets by their social properties and relate these for example to innovation and imitation effects (the p and q parameters) of the Bass diffusion model and hence find different impacts of the structural properties on future market share development.

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